TABLE 2. CATION EXCHANGER MEMBRANE WITH $M^+ (M^{*+})$, R^-, AND W

	$\alpha_i = 0$		$\alpha_i = 1$		$\alpha_i > 0$			
Operation	$\mathcal{D}_{M^+I}$	$\mathcal{D}_{WI}$	$\mathcal{D}_{M^+m}$	$\mathcal{D}_{Wm}$	$\mathcal{D}_{M^+, \text{eff}}$	$\mathcal{D}_{W, \text{eff}}$	α_{M^+}	α_W
Self-diffusion	3.50×10^{-6}	—	3.50×10^{-6}	—	3.46×10^{-6}‡	5.35×10^{-6}	—	1.02
	3.51×10^{-6}¶		3.51×10^{-6}¶		3.47×10^{-6}¶			
Conductivity	5.66×10^{-6}	—	2.59×10^{-6}	—	3.47×10^{-6}	5.35×10^{-6}	0.71	1.00
Streaming current	—	2.43×10^{-4}	4.92×10^{-6}	5.35×10^{-6}	3.47×10^{-6}	5.35×10^{-6}	0.706	1.00

¶ $\bar{D}_{M^{*+}M^+}$ assumed equal to infinity.
‡ with $x_{M^{*+}} = 0.0055$.

following definition will be used:

$$\mathbf{N}_i = -c\, D_{im} \nabla X_i + \alpha_i x_i \sum_{j=1}^{n} \mathbf{N}_j \qquad (1a)$$

$$= -c\, D_{im} \nabla X_i + \alpha_i\, c_i\, \mathbf{v}^* \qquad (1b)$$

Here

$$\nabla X_i = \nabla x_i + x_i \nabla \ln f_i + \frac{x_i \nu_i F}{RT} \nabla \phi + \frac{x_i}{RT}\left(\bar{V}_i - \frac{M_i}{\rho} \right) \nabla p \qquad (2)$$

Equation (1) is convenient for several reasons.

1. It is consistent with the accepted form of Fick's first law for the limiting case of a binary system (2) with only concentration diffusion when $\nabla X_i = \dfrac{\partial \ln f_i x_i}{\partial \ln x_i}$ and $\alpha_i = 1$.

2. It allows for the sometimes important effects of convection, through inclusion of the last term on the right.

3. The use of the molar-average velocity, $\mathbf{v}^*$, rather than the mass-average velocity, $\mathbf{v}$, of Helfferich (*8*) results in a simpler set of flux expressions.

4. The introduction of α_i, usually considered to be unity or zero in existing correlations of experimental data, will be shown to be useful in the discussion below.

A completely analogous set of equations may be written in terms of mass rather than molar quantities, and it is in such an equation that the mass-average velocity is best used.

It only remains to put the more reliable flux equations of irreversible thermodynamics in a similar form. To do this one begins with the generalized Stefan-Maxwell equation (*10*):

$$\nabla X_i = \sum_{j=1}^{n} \frac{x_i x_j}{\bar{D}_{ij}} (\mathbf{v}_j - \mathbf{v}_i) \qquad (3a)$$

$$= \sum_{j=1}^{n} \frac{1}{c\, \bar{D}_{ij}} (x_i \mathbf{N}_j - x_j \mathbf{N}_i) \qquad (3b)$$

Equations (3*a*) and (3*b*) are equivalent to the expressions introduced earlier by Speigler (*13*)* but are more convenient for the present purpose. First, they are of the same form as the Stefan-Maxwell equations for ideal gases, thus permitting immediate use of the pseudo-binary expressions that have been developed for these better-understood systems. Scond there is reason to believe (*10*) that the $\bar{D}_{ij}$ in Equations (3*a*) and (3*b*) are less concentration dependent than most other phenomenological coefficients now in use (see Table 1).

There are many ways to force Equations (3*a*) and (3*b*) into the form of Equation (1), but three seem especially promising.

The method used most frequently is

(*Continued on page 185*)

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* The friction coefficients defined by Spiegler, which are also consistent with the principles of irreversible thermodynamics, are given by

$$\frac{RT \nabla X_i}{x_i} = \sum_{j=1}^{n} F_{ij} (\mathbf{v}_j - \mathbf{v}_i)$$

so that

$$F_{ij} = \frac{RT x_j}{\bar{D}_{ij}}$$

with

$$x_i F_{ij} = x_j F_{ji}$$

Distillation Studies — Viscous Drag on a Gas Bubble Rising in a Liquid

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This study forms part of an investigation into the fluid mechanics of a perforated plate distillation column at its minimum load. In particular, the seal point (*6*), at which stage liquid is just flowing over the downcomer weir, is considered. Here the gas is bubbling, singularly or otherwise, through some of the holes, while liquid is dumping through the others. One equation of motion can be obtained from a momentum balance on a rising bubble:

(Net Buoyance Force + Excess Pressure + Rate of Loss of Gas Momentum) = (Viscous Drag + Surface Tension Drag + Inertial Force of the Rising Bubble)

For a single gas bubble, the viscous drag term is usually represented by

Drag Force = Drag Coefficient × Inertial Stress × Projected Area

or

$$F_D = C_D \cdot \frac{1}{2} \rho_L U^2 \cdot \frac{\pi}{4} D_e^2$$

The drag coefficient may be correlated experimentally by

$$C_D = a / N_{Re}$$

where

$$N_{Re} = D_e U \rho_L / \mu_L$$

PREVIOUS WORK

Datta (*1*) and Gorring (*3*) have studied various proposed relations for the value of a:

Stokes' equation:
$a = 24$

Moore's equation:
$a = 32$

Hadamard's equation:

$$a = 24 \left(\frac{2}{3} \frac{\mu_L + 3\mu_G}{\mu_L + 3\mu_G} \right)$$

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INFORMATION RETRIEVAL

(*Continued from page 186*)

Thermodynamics of mixed-gas adsorption, Myers, A. L., and J. M. Prausnitz, **A.I.Ch.E. Journal, 11,** No. 1, p. 121 (January, 1965).

Key Words: Mixture Adsorption Equilibria-2, Adsorption-1, Thermodynamics-10, Ideal Solution-10, Silica Gel-9, Activated Carbon-9, Methane-9, Ethane-9, Ethylene-9, Carbon Dioxide-9, Carbon Monoxide-9, Oxygen-9, Propane-9, Propylene-9.

Abstract: A simple technique is described for calculating the adsorption equilibria for components in a gaseous mixture, using only data for the pure-component adsorption equilibria at the same temperature and on the same adsorbent. The proposed technique is based on the concept of an ideal adsorbed solution, and, with classical surface thermodynamics used, an expression analogous to Raoult's law is obtained. Predicted isotherms give excellent agreement with experimental data for methane-ethane and ethylene-carbon dioxide on activated carbon and for carbon monoxide-oxygen and propane-propylene on silica gel. The simplicity of the calculation, which requires no data for the mixture, makes it especially useful for engineering applications.

Lateral transport in a fluidized-packed bed: Part I. solids mixing, Gabor, J. D., **A.I.Ch.E. Journal, 11,** No. 1, p. 127 (January, 1965).

Key Words: Solids Mixing-7, Diffusivity-7, Fluidization-8, Particles-6, Fluidized-Packed Bed-8, Velocity-6, Minimum Fluidization Velocity-6, Packing-6, Bed Expansion-6, Drag Force-6, Gas Bubbles-6.

Abstract: The lateral solids mixing diffusivities of the fluidized particles in a fluidized-packed bed depend upon the average particle velocity and the size of the fixed packing. The average particle velocity was related to the fluidizing gas velocity and the minimum fluidization velocity. This relationship was based on an analysis of particle movement caused by drag forces associated with gas bubbles rising through the fluidized bed.

Lateral transport in a fluidized-packed bed: Part II. heat transfer, Gabor, J. D., B. E. Stangeland, and W. J. Mecham, **A.I.Ch.E. Journal, 11,** No. 1, p. 130 (January, 1965).

Key Words: Effective Thermal Conductivity-7, Fluidized-Packed Bed-8, Heat Transfer-9, Solids Mixing-6, Thermal Diffusivity-7, Packing-6, Fluidization-9, Velocity-6, Minimum Fluidization Velocity-6, Diffusivity-7.

Abstract: Effective thermal conductivities for lateral heat transfer were measured in a fluidized-packed bed. An analysis of the mechanism of heat transfer was made by comparison with previously determined rates of solids mixing. A general correlation was made of the fluidized-packed bed thermal diffusivities with the size of the fixed packing and the minimum fluidization velocity in a manner similar to the correlation for lateral solids mixing diffusivities in a fluidized-packed bed.

Analysis of phase-boundary motion in diffusion-controlled processes: Part I, Griffin, J. R., and D. R. Coughanowr, **A.I.Ch.E. Journal, 11,** No. 1, p. 133 (January, 1965).

Key Words: Moving-Boundary Problem-8, Diffusional Process-8, Heat and Mass Transfer-8, Ablation-8, Riemann-Volterra Integration-10, Green's Function-10, Intermediate Integral-10, Finite-Difference Approximation-10, Differential Analyzer-10, Analogue Computer-10.

Abstract: Three general methods are developed for solving moving-boundary problems which are governed by diffusional processes such as heat and mass transfer. Examples of such problems include melting, evaporation, and ablation. A method based upon a Riemann-Volterra integration of the diffusion equation leads to nonlinear integrodifferential equations for the boundary motion that are in terms of definite integrals involving Green's functions. An analytical method, which is more convenient for problems involving phase motion, is based on the method of intermediate integrals. A numerical method based on finite difference approximations is implemented on the differential analyzer (analogue computer).

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Boussinesq's equation:

$$a = 24\left(\frac{\epsilon + (2\mu_L + 3\mu_G)\, r}{\epsilon + (3\mu_L + 3\mu_G)\, r}\right)$$

Allen's equation:

$$a = \frac{9}{2}\left(\frac{\beta r + 3\mu_L}{\beta r + 2\mu_L}\right)$$

The last three are difficult to use in calculations. Gorring (*3*), using the minor diameter in calculating C_D and the major diameter in calculating N_{Re} on published as well as his own data, obtained an improved correlation but one which was still not satisfactory for prediction.

During preparation of this communication it was pointed out to the authors that the problem was being studied by Moore (*5*) and a similar conclusion about the drag force had been reached by Levich (*4*). Nevertheless it is felt that the result obtained independently by the authors should be communicated as a footnote to the work of Gorring (*3*).

THEORETICAL STUDY OF THE VALUE OF "a"

Derivation of the relation proposed here, namely

$$C_D = 48/N_{Re} \tag{1}$$

is given in detail so that the various assumptions may be indicated.

The kinetic energy, E, at any time, t, of the liquid in a region R' is, in Cartesian form

$$E = \frac{1}{2}\int_{R'} \rho_L\,(u^2 + v^2 + w^2)\,dR'$$

It can be shown (*7*) that for a Newtonian fluid the rate of loss of E is

$$\frac{dE}{dt} = I_{R'} = -\mu_L \int_{R'} \left\{ 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left[\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2\right]\right\} dR' \tag{2}$$

if one assumes that the rate of doing external work = 0, and the rate of loss of E owing to normal boundary stresses = 0. The second assumption is reasonable if no boundary layer exists for a gas bubble in a liquid, or if a boundary layer does exist, effects of normal boundary stresses are small in order of magnitude in comparison with losses of E owing to shear or tangential stresses.

Equation (2) may be integrated if the velocity gradients are known. It seems plausible that near the bubble surface the velocity potential, ϕ, for irrotational flow, gives as a first approximation a uniform field. From this the velocity gradients may be derived. Alternatively, other assumptions must be made about the velocity gradients. The rest of this communication shows that if the assumption stated above is made, the result agrees well with the Gorring correlation (3) of experimental data.

To find ϕ, the assumption of a spherical bubble is made. For continuity in the liquid phase, taken as incompressible

$$\nabla \cdot \mathbf{u} = 0$$

With boundary conditions

$$\mathbf{u} \rightarrow U \cos\theta \qquad \text{as } r \rightarrow \infty$$

$$\mathbf{u} \cdot \mathbf{n} = 0 \qquad \text{at } r = De/2$$

For irrotational motion

$$\mathbf{u} = -\nabla\phi$$

so that

$$\nabla^2 \phi = 0$$

Using the two boundary conditions, ϕ is shown to be by classical methods (7)

$$\phi = -U r \left(1 + \frac{R^3}{2\, r^3}\right) \cos\theta \qquad (3)$$

From Equation (3), the two velocity components for axisymmetry are

$$u_r = -\frac{\partial\phi}{\partial r} = U\left(1 - \frac{R^3}{r^3}\right)\cos\theta$$

and

$$u_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\left(1 + \frac{R^3}{2r^3}\right)\sin\theta$$

The nine velocity gradients may now be derived:

$$\left.\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{\partial u_\theta}{r\partial\theta} + \frac{u_r}{r} = -\frac{3UR^3}{2r^4}\cos\theta \\
\frac{\partial u}{\partial y} &= \frac{u_\theta}{r \sin\theta\, \partial\omega} - \frac{u_\omega \cot\theta}{r} = 0 \\
\frac{\partial u}{\partial z} &= \frac{\partial u_\theta}{\partial r} = \frac{3UR^3}{2r^4}\sin\theta
\end{aligned}\right\} \qquad (4a)$$

$$\left.\begin{aligned}
\frac{\partial v}{\partial x} &= \frac{\partial u_\omega}{r\partial\theta} = 0 \\
\frac{\partial v}{\partial y} &= \frac{\partial u_\omega}{r \sin\theta\, \partial\omega} + \frac{u_r}{r} + \\
&\qquad \frac{u_\theta \cot\theta}{r} = \frac{-3\, U\, R^3}{2r^4}\cos\theta \\
\frac{\partial v}{\partial z} &= \frac{\partial u_\omega}{\partial r} = 0
\end{aligned}\right\} \qquad (4b)$$

INFORMATION RETRIEVAL

(*Continued from page 188*)

Generation of waves in the concurrent flow of air and a liquid, Cohen, Leonard S. and Thomas J. Hanratty, **A.I.Ch.E. Journal, 11,** No. 1, p. 138 (January, 1965).

The initiation of waves at the interface of a concurrent air-liquid flow has been studied. The disturbance in the velocity field in the air caused by waves at the interface gives rise to pressure and shear stress variations over the wavy surface which transmit energy to the liquid film. If the energy transmitted by these mechanisms is larger than the viscous dissipation in the liquid, the waves are generated. Agreement with experimental measurements of the critical air velocities, wave lengths, and wave velocities is obtained using the model of the gas flow proposed by Miles and Benjamin.

Key Words: Waves-8, Stability-9, Interface-8, Two-Phase Flow-8, Wave Lengths-7, Wave Velocities-7, Air Velocity-6, Liquid Viscosity-6, Liquid Velocity-6, Critical Velocity-7, Initiation of Waves-7.

The effect of shape and density on the free settling of particles at high Reynolds numbers, Christiansen, E. B., and Dee H. Barker, **A.I.Ch.E. Journal, 11,** No. 1, p. 145 (January, 1965) .

Key Words: A. Drag Coefficients-8, 9, 7, Determination-8, Cylinders-9, Prisms-9, Disks-9, Spheres-9, Air-5, Water-5, Reynolds Number-6, Density-6, Lift-6, Fluids-9, Particles-9, Shape-6, Fluid Dynamics-8, Settling-8, Metals-9, Sedimentation-8, Viscosity-6, Aluminum-9, Lucite-9, Magnesium-9, Oils-5, Theoretical-, Experimental-, Turbulent-, Glucose-5, Rotation-6, Moment of Inertia-6, Oscillation-6, Photoelectric Cells-10, Field Force-6, Dimensional Analysis-10, Physical Properties-6.

Abstract: Terminal velocity drag coefficients C_D were determined for cylinders, prisms, disks, and spheres in air, water, oil, and glucose. C_D is a function of particle and fluid densities, shape, and Reynolds number. The data for N_{Re} = 1,000 to 300,000 are correlated by considering C_D a function of particle moment of inertia and the rotational moment generated by circulation.

Analysis of phase boundary motion in diffusion-controlled processes: Part II. Application to evaporation from a flat surface, Griffin, J. R., and D. R. Coughanowr, **A.I.Ch.E. Journal, 11,** No. 1, p. 151 (January, 1965).

Abstract: The details involved in using the three general methods of Part I for solving moving-boundary problems are illustrated by applying them to the evaporation of a liquid from a flat surface into a vapor phase of infinite extent. The process is controlled by mass diffusion of the evaporated species through a vapor consisting of the diffusant and a gas, such as air, which is insoluble in the liquid.

Key Words: Moving-Boundary Problem-8, Diffusional Processes-8, Evaporation From a Flat Surface-8.

Thermal pressure applied to the prediction of viscosity of simple substances in the dense gaseous and liquid regions, Lennert, David A., and George Thodos, **A.I.Ch.E. Journal, 11,** No. 1, p. 155 (January, 1965).

Key Words: Thermal Pressure-1, Residual Viscosity-1, Critical Compressibility Factor-1, Viscosity-8, Dense Gaseous State-9, Liquid State-9, Argon-10, Krypton-10, Xenon-10.

Abstract: The reduced thermal pressure-temperature ratio has been used to relate the residual viscosity modulus for argon, krypton, and xenon into a unique relationship applicable for the dense gaseous and liquid regions. For these monatomic substances the critical compressibility factor is $z_c = 0.291$. Values of $(\partial P_R/\partial T_R)_{\rho R}$ vs. $(\mu - \mu^*)\xi$ on log-log coordinates produced a linear relationship. For these simple substances, this relationship was used to predict viscosities with an average deviation of 3.0% for fifty eight experimental values. This relationship was also applied for the prediction of viscosities for nitrogen, oxygen, and carbon dioxide.

The approach developed in this study merits further examination with several additional substances. The lack of adequate thermal pressures in the dense gaseous and liquid regions of substances other than argon limits the use of this study to substances having critical compressibility factors $z_c = 0.291$.

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$$\left.\begin{aligned}\frac{\partial w}{\partial x} &= \frac{\partial u_r}{r\partial\theta} - \frac{u_\theta}{r} = \frac{3UR^3}{2r^4}\sin\theta\\ \frac{\partial w}{\partial y} &= \frac{\partial u_r}{r\sin\theta\,\partial\omega} - \frac{u_\omega}{r} = 0\\ \frac{\partial w}{\partial z} &= \frac{\partial u_r}{\partial r} = \frac{3U\,R^3}{r^4}\cos\theta\end{aligned}\right\}\quad (4c)$$

If Equations (4*a*), (4*b*), and (4*c*) are substituted into Equation (2), one obtains

$$I_{R'} = -\,9\mu_L\,U^2R^6\int_{R'}\frac{1}{r^8}$$

$$(1 + 2\cos^2\theta)\;dR' \qquad (5)$$

The region of integration is (R, ∞) for r, over the spherical surface for which an elemental area is $dA = 2\pi\,r^2\sin\theta\cdot d\theta\cdot dr$, and $(0, \pi)$ for θ, so that

$$\begin{aligned}-\,F_DU &= \frac{dE}{dt} = I_{R'}\\ &= -\,18\mu_L\,U^2R^6\int_R^\infty\frac{dr}{r^6}\int_0^\pi\\ &\qquad(1 + 2\cos^2\theta)\,\sin\theta\,d\theta\\ &= -\,18\mu_L\,U^2\,R^6\cdot\frac{1}{5R^5}\cdot\frac{10}{3}\\ &= -\,12\,\pi\,\mu_L\,U^2R\end{aligned}$$

from which Equation (1) follows directly.

This equation has now been derived without making a specific assumption about the value of $p_{r\theta}$ at $r = De/2$, and will then be valid whether slip exists ($p_{r\theta} = 0$) or not. It can be shown (*5*) however, that if the contributions to the drag of the boundary layer and wake are neglected, the errors in C_D as predicted by Equation (1) are of the order of $(N_{Re})^{-3/2}$ only.

COMPARISON WITH EXPERIMENTAL CORRELATION

The collection of data by Haberman and Morton on the rise of air bubbles in liquids was presented graphically by Gorring (*3*). The fifty data points were read from the graph and analyzed statistically. If one takes the relation between C_D and N_{Re} to be $C_D = a/(N_{Re})^b$, and if the errors in determining N_{Re} are small compared to those in determining C_D, one finds, using well known methods (*2*)

$$C_D = 28/(N_{Re})^{0.92} \qquad (6)$$

An analysis of variance indicated that the regression was highly significant (exceeded the 99% confidence level). Lines corresponding to 95% confidence limits are drawn on Figure 1 which presents these comparisons graphically, the two error limit lines being approximately linear.

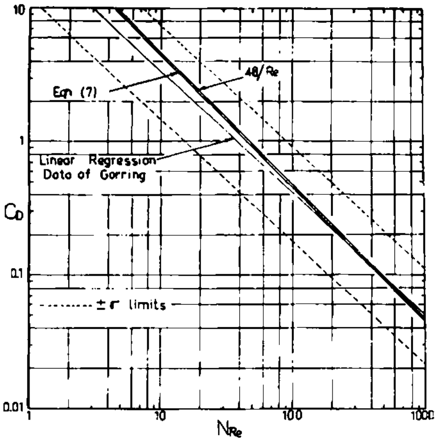

Fig. 1. Air bubbles rising in various liquids taken from data of Gorring and Katz (3).

The line corresponding to the equation derived here [Equation (1)] is also shown and may be seen to lie well within the confidence limits. Particularly at the higher Reynolds numbers, it represents the data as well as the best line, Equation (6).

The index, —0.92, of N_{Re} for the regression line does not differ significantly from — 1.00, so that the experimental data may be represented by the relation

$$C_D = a/N_{Re} \qquad (7)$$

The best statistical estimate of a in Equation (7) was found to be 46.7 with ±3.5 confidence limits, so that it does not differ significantly from 48.

It follows that Equation (1), which has been derived theoretically, represents the data as well as purely empirical relations. Figure 2 shows a comparison between the values of C_D calculated from the regression given by Equation (6) and the values predicted by Equation (1) for N_{Re} = 10, 20, 40, 50, 60, 80, 100, 200, 400, and 500.

CONCLUSION

For the Reynolds number range 10 to 500, the viscous drag on a rising

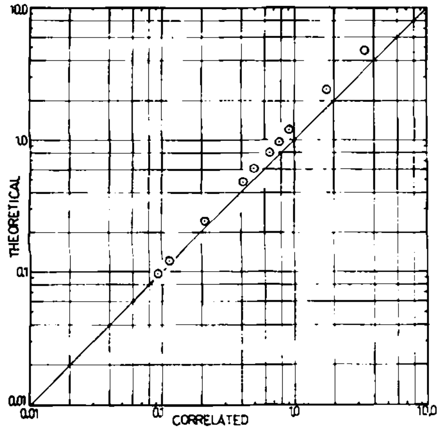

Fig. 2. Comparison of values of drag coefficients. Gorring correlation vs. (48/Re) for Re = 10, 20, 40, 50, 60, 80, 100 200, 400, 500.

(Continued from page 187)

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INFORMATION RETRIEVAL

(Continued from page 190)

The influence of mass transfer on a countercurrent liquid-liquid fluidized system, Dunn, I., Leon Lapidus, and J. C. Elgin, **A.I.Ch.E. Journal, 11,** No. 1, p. 158 (January, 1965).

Key Words: A. Mass Transfer-6, Holdup-7, Fluids-9, Liquid-Liquid Fluidized Systems-9, Countercurrent-0, Extraction-4, Solute-6, Droplets-9, Coalescing-7, Experimental-, Rate-6, Flow-9, Separation-4. B. Mass Transfer-6, Extraction-4, Solute-9, Operation-7, Columns (Process)-9, Separation-4. C. Holdup-8-7, Methyl Isobutyl Ketone-5, Water-5, Acetic Acid-1, Dodecyl Sodium Sulfate-6, Surface Active Agents-6, Aliphatic Compounds-1, Carboxylic Acid-1, Ketones-5. D. Holdup-8-7, Toluene-5, Water-5, Acetone-1, Aromatic Compounds-5, Dodecyl Sodium Sulfate-6, Surface Active Agents-6, Aliphatic Compounds-1, Hydrocarbons-5, Ketones-1.

Abstract: The effect of mass transfer on holdup of countercurrent fluidized systems and the effect of solute on droplet coalescence were determined experimentally. The effect of solute transfer on operation of extraction columns is discussed. Data were taken on two systems, methyl isobutyl ketone-water and toluene-water with acetic acid and acetone solutes, respectively. Dodecyl sodium sulfate was added to some of the runs to determine the effect of surface active agent.

The thermal conductivity and diffusivity of gases for temperatures up to 10,000°K., Mathur, G. P., and George Thodos, **A.I.Ch.E. Journal, 11,** No. 1, p. 164 January, 1965).

Key Words: High Temperatures-6, Self-Diffusivity-8, Mutual Diffusivity-8, Thermal Conductivity-8, Generalized-9, Dimensional Analysis-9, Critical Constants-10.

Abstract: A generalized method has been developed for the prediction of the diffusivity and thermal conductivity of nonpolar gases for temperatures up to 10,000°K. This method is applicable to monatomic gases and to diatomic gases existing in their molecular and dissociated states. Dimensional analysis has been used to obtain relationships for self- and mutual diffusivities and for the translational contribution to the thermal conductivity of these substances. For the internal contribution to thermal conductivity, a modified Eucken-factor relationship is presented. Furthermore, relationships have been developed for the calculation of these transport properties for the equilibrium mixture of a dissociating gas.

Particle flow patterns in a fluidized bed, Marsheck, R. M., and A. Gomezplata, **A.I.Ch.E. Journal, 11,** No. 1, p. 167 (January, 1965).

Key Words: Flow Patterns-2, 8, Solid Particles-0, Fluidized Bed-9, Thermistor-0, Anemometer-0, Probe-10, Silica-Alumina-10, Velocity-6, Air-0, Bed Height-6, Radial-0.

Abstract: Solid particle flow patterns in a fluidized bed were studied with a modified thermistor anemometer probe. Radial flow patterns were found not to be symmetrical with the axis of the fluidized bed but were similar at angles of 60 deg. The percentage of the cross-sectional area of the bed through which the particle flow is upward was found to be 60%. The principal factors affecting the flow pattern were the superficial air velocity and the ratio of the distance above the support plate to the fluidized bed height.

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bubble can be satisfactorily calculated despite the assumption of spherical bubbles when their real shape is that of an oblate spheroid.

ACKNOWLEDGMENT

Helpful discussions with V. T. Buchwald of the Department of Applied Mathematics, and R. I. Tanner of the Department of Mechanical Engineering of the University of Sydney, Sydney, Australia, are acknowledged.

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NOTATION

a = constant
b = constant
C_D = drag coefficient in clear liquid based on D_e
D_e = equivalent diameter (of sphere of same volume)
E = kinetic energy
F_D = drag force
I_R' = volume integral
$p_{r\theta}$ = tangential stress
R = $(D_e/2)$
R' = region of liquid investigated
r = bubble radius
t = time
U = velocity of rising bubble relative to liquid
u_r, u_θ, u_ω = velocity components, spherical form
u, v, w = velocity components, Cartesian form
$\mathbf{n}$ = unit vector, normal to bubble surface
N_{Re} = $(D_e\, U\, \rho_L/\mu_L)$
β = coefficient of sliding friction
ϵ = coefficient of surface viscosity
θ = angle between r and the vertical
μ_G = gas viscosity
μ_L = liquid viscosity
ρ_L = liquid density
σ = standard deviation
ϕ = velocity potential
ω = angle in horizontal plane
∇ = the Laplacian operator

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